TRADITION AND INNOVATION IN THE CONSOLIDATION OF RIB VAULTS

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ABSTRACT
Palazzo Stradiotti, in Cremona, is a noble residence of eighteenth century. After a long period of disuse, the Stauffer Academy decided to transform the building in a Music House. Therefore, it was necessary to adequate the structures to the new loads, as established by the Italian normative. In this paper, one particularly vulnerable rib vault of the building has been analyzed, subjected to vertical loads, in order to evaluate the bearing capacity and to define a suitable consolidation method. A no linear analysis was carried out on a FE model of the vault, performed by using Abaqus 6.10. It demonstrated that the existing vault can support an increment of symmetrical loads, but it failed if the loads are not symmetrically applied. Therefore, four different consolidation methods were analyzed, to improve the bearing capacity. In detail the traditional technique of the *frenelli* was adopted.The results obtained by using concrete and masonry *frenelli*, differently positioned, were compared. The comparison proved that, in spite of the lower stiffness, a significant improvement of the structural behavior can be obtained also by using masonry *frenelli*.

1. INTRODUCTION

The renewed interest for restoration work on historic existing buildings, the availability of new diagnostic tools and the recent focus on the structural safety of historical constructions under seismic actions, required a deeper study of the theories and methods of intervention on curvilinear structural systems. In spite of the high number of research work on the static behavior of arches, masterly summarized by J. Heyman [3, 4], more work is needed on the strengthening techniques of masonry arches and vaults, included in historical building.
Among the possible interventions that can be adopted to avoid the formation of hinges, there are: the use of FRP on the extrados or on the intrados or the traditional reinforcing concrete upper layer. Although they are structurally efficient systems, in many cases they appear invasive, non-reversible and passive. Furthermore, in the presence of painted sur-

face, the complete covering of the extrados of vaults and domes by means of concrete or, in general, using incompatible material layers, can limit the transpiration of the material, so that frescoes can be damaged.
In 1997 one of the authors proposed a reversible and active system to reinforce masonry arches, called "RAM - Reinforced Arch Method" [5]. In the last twenty years this method has been widely tested and used by the author, prof. Jurina, for the consolidation of precious monuments such as Cremona Cathedral [6], Santa Caterina Church in Lucca [7], Colorno Cathedral (Italy) [8], each one fitting different geometries.
In the present paper, an alternative more traditional method, based on realizing *frenelli* on the vault, is applied for consolidation of a particularly ribbed vault of *Palazzo Stradiotti*, in Cremona. The use of different materials and positions of the *frenelli* was evaluated by performing FE numerical analyses, with the aim to establish the most effective strengthening method.

2. *PALAZZO STRADIOTTI*

Palazzo Stradiotti is an eighteenth-century residential building located in Cremona and from long time abandoned. It consists of four constructions placed inside a rectangular Romantic garden: a main building with a horseshoe shape, a smaller two-floor building and a rectangular and an octagonal temple. The main building is supported by a masonry vertical structure 40 cm thick. The horizontal structures consist of vaults and wooden roof, while the roof is gabled.

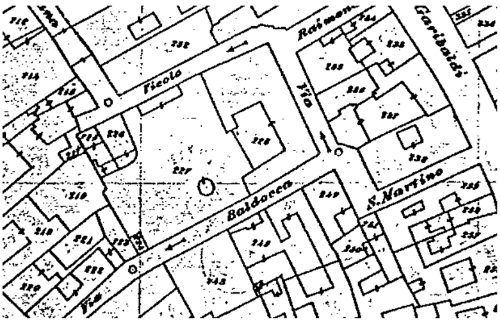


Figure 1. Cadastral plan

In 2016 *Palazzo Stradiotti* has been acquired by the *Fondazione/Centro di Musicologia Walter Stauffer* as location of the Stauffer Academy, which host advanced courses for chamber musicians.
Due to the change of use, the building required architectural and structural intervention. From the structural point of view, it was necessary, first of all, to adequate the horizontal structures to the loads prescribed from the Italian rules. [10].

3. NO-STRENGTHENED RIBBED VAULTS

The most vulnerable vaults of *Palazzo Stradiotti* are the ribbed vaults covering the portico at the the ground floor and the corresponding rooms of the first floor. Their vulnerability is due, first of all, to the typology in fact ribbed vaults lie only on four punctual support. In addition, the ribbed vaults of *Palazzo Stradiotti* are quite skene and characterized by a long span. The central ribbed vault of the first floor was chosen, among the others, to carry out the analysis. It was chosen for two reasons. First, the central one was selected because it has the maximum span. Second, the vaults of the first floor were more difficult to consolidate, due to the limited height which the repair intervention could reach. Figure 2a shows the plan of the first floor with the indication of the selected vault. Figure 2b shows the section of the vault. It has a rectangular plan of dimensions equal to 5.8 x 3.7 m. The distance between the springer and the keystone is equal to 1.25 m.

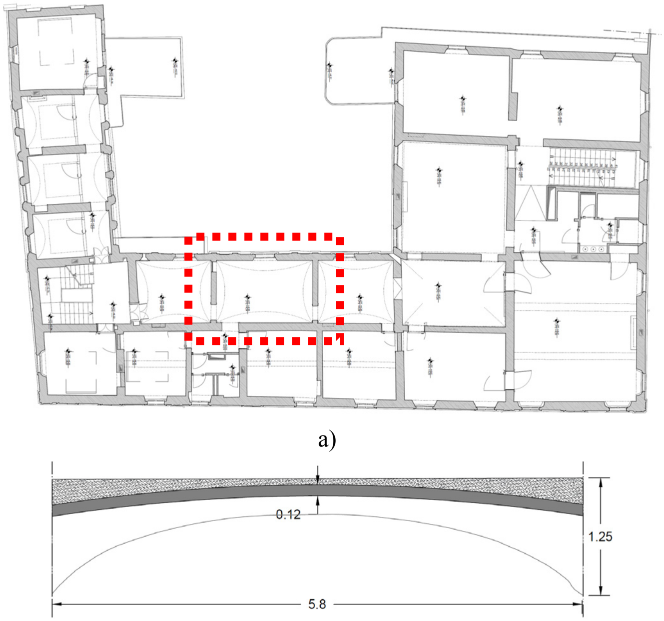


Figure 2. a) Plan of the first floor; b) Section of the selected vault

On the basis of the endoscopy tests conducted on other vaults of the building, the vault was assumed as a masonry layer equal to 12 cm and a filling made of incoherent material.

3.1. FE model

A FE model of the vault was performed, by using the software Abaqus 6.10, and it is shown in Figure 3a. The vault was modeled as a shell, having the thickness of the masonry layer (12 cm). Quadrilateral shell elements were used to mesh the model. The average size of the

elements is equal to 0.1 m,
The presence of the filling was simulated just applying a distributed load on the shell, as its material gives a poor contribution to the resistance of the vault. The intensity of the distributed load was differentiated according to the partition reported in Figure 3b, which takes into account the different height of the filling along the vault. Zone 1 has a distributed load equal to 30 kN/m^3, zone 2 equal to 20 KN/m^3 , zone 3 equal to 15 kN/m^3, zone 4 equal to 10 kN/m^3.
The model was assumed simply supported at the four corners. In order to avoid tension concentration at one point, the support constraints were assigned also to the six adjacent joints.

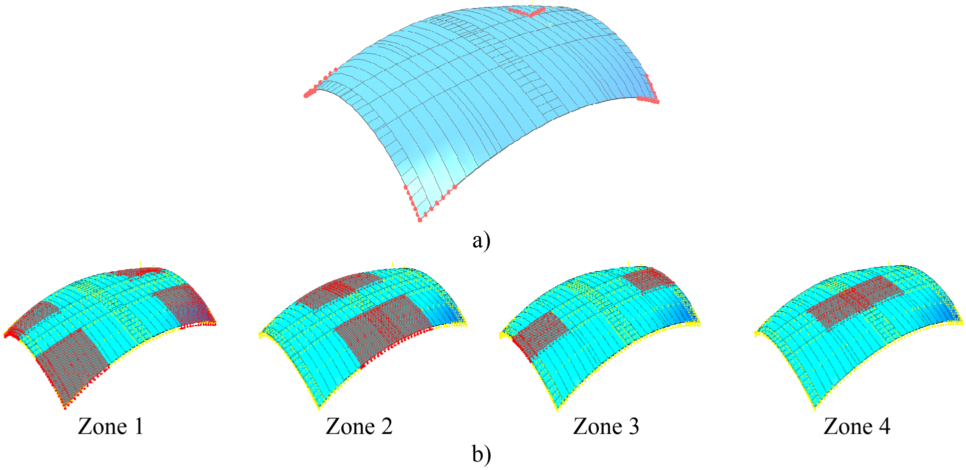


Figure 3. a) Model of the no-reinforced vault; b) Distribution of the filling loads

The mechanical characteristics of the model were extracted from the results of the flat jacks, carried out during the diagnostic campaign performed in june and july 2016. The assigned properties are reported in Table 1.

Density (daN/m^3)	Young's modulus (MPa)	Poisson ratio (-)	Compressive strength (MPa)	Tensile strength (MPa)
1800	907	0.3	2	0.1

Table 1. Material properties

The value of the tensile strength, which was not provided from the experimental test, was assigned according to similar studies on masonry constructions. Tensile strength was considered equal to 5% of compressive strength [1, 9].

3.2. Non linear analysis

The described model was analyzed by means of a non linear static analysis. In detail, the

non linearity of the material was taken into account. To model brittle structures, Abaqus provides the "concrete smeared model". It is a fixed multi-crack model based on a simple yield surface with isotropic hardening and associated flow when the state of stress is predominantly compressive, and uses damaged elasticity to account for the cracking, the occurrence of which being defined by a so-called "crack detection surface". This failure surface is assumed to be a simple Coulomb line written in terms of the first and second stress invariant. The concrete model basically requires: 1) the stress–strain curve in compression to be defined in tabular form as a function of plastic strain, 2) the shape of the failure surface via the "failure ratios"; 3) the post-cracking tensile behaviour defined by the "tension stiffening" option. The tension stiffening implies that, as the tensile strength is reached, the σ - ε curve goes to 0 following a straight or curve line, according to the user choice. This last feature actually makes no sense for masonry, but a small amount of tensile resistance should be anyway provided to avoid numerical instability problems.
The non linear analysis of the ribbed vault was carried out by adopting the following data. As stress-strain law in compression, the curve of Figure 4 was used, obtained from the Turnsek-Cacovic relation. The failure ratios, defined on the basis of sensitivity analysis performed by Giordano et al. in [2], are: R1=1.16; R2=0.05; R3=1.33; R4=0.3. In tensile regime, the stress-strain relationship is linear until the tensile strength is reached.
Then a linear tension stiffening function is adopted, and it was defined by assigning a value of the ultimate strain equal to 0.003.

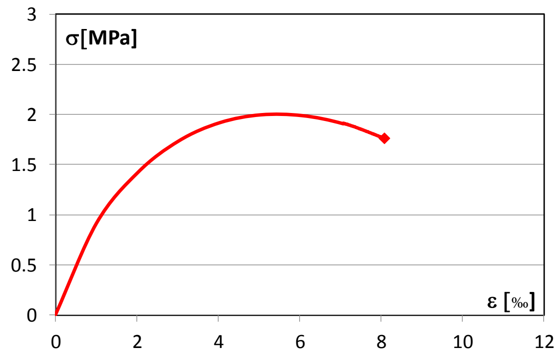


Figure 4. Constitutive law in compression

The analysis was conducted for four different load cases, schematized in Figure 5. In CASE 1, the vault was loaded only by the self-weight and the weight of the filling; in CASE 2 the vault was loaded also by live loads, distributed on the whole vault, equal to 2 kN/m^2 ; in CASE 3 the live load was applied only on half vault; in CASE 4 the live load was increased to 4 kN/m^2 , which is the live load required from Italian rules for the new use of the building.

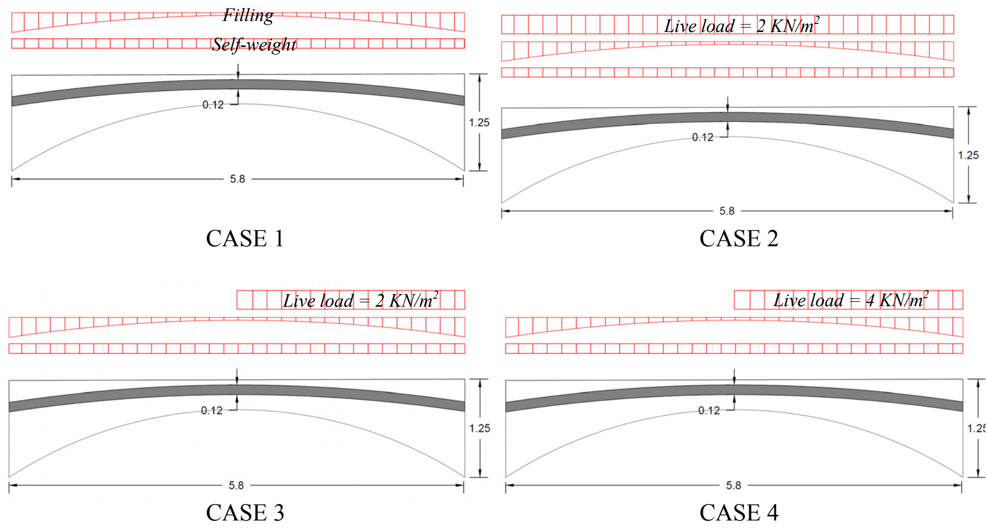


Figure 4. Load cases

3.3. Results

The results of the non linear analysis for the 4 considered load-cases are shown in Table 2, where, for each load-case, the following output are listed: 1) the stress state in terms of the component σ_{11}; 2) the stress state in terms of the component σ_{22}; 3) the displacements (D).
CASE 1 shows a predominant compressive stress state, except of the central zones of the perimeter arches. The average value of the compressive stresses is approximately equal to 0.2 MPa, even if concentrations of stress were found at the passage between the constraint joints and the free ones. The tensile stress is on average equal to 0.01 MPa, acceptable for masonry, and only at the keystone of the perimeter arches it reaches the limit value of 0.1 MPa. The maximum displacement was found at the keystone of the longer perimeter arch and it is equal to 4.5 mm. It can be deduced that the vault was able to support the self-weight and the weight of the infills.
CASE 2 shows results qualitatively similar to the previous case. Due to the increase of the loads, the tensile zone is larger and the average value is higher. Moreover, it can be observed that the maximum displacement is equal to 7.6 mm, significantly higher than the previous case. However, the masonry vault is still well conformed to support this kind of load.
CASE 3 is the first no-symmetrical load case. The results shows a consequent no symmetrical stress state. By observing the component σ_{11}, it can be noted that the tensile zone is significantly larger and it goes from the middle zone of the perimeter transversal arches, to the middle zone of the longitudinal ones. Similarly, the component σ_{22} gives tensile stresses not only in the middle of longitudinal arches, as it was found in the previous cases, but small tensile zone were found also at the keystone of one transversal arch. As expected, the behavior of the vault for a no-symmetrical load get worse, and a large part suffers probable damage. However, maximum displacement in this case is equal to 6.3 mm, lower than case

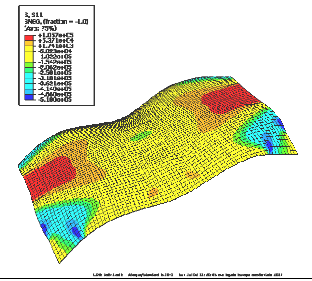
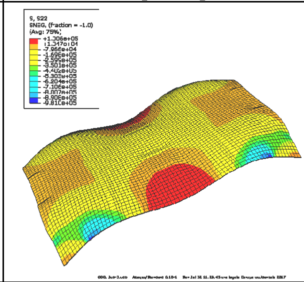
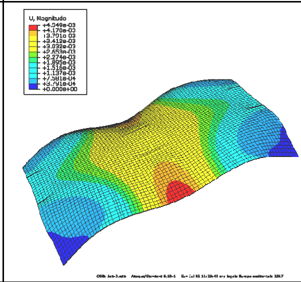
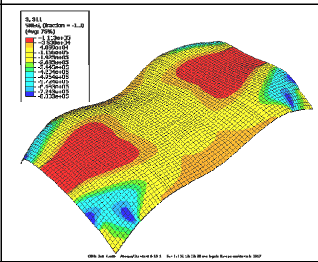
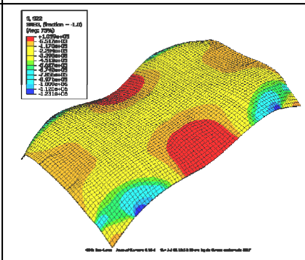
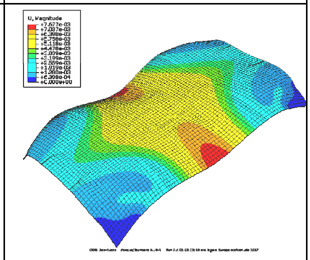
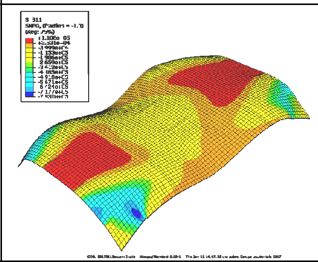
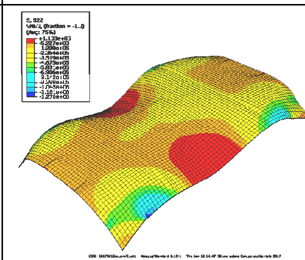
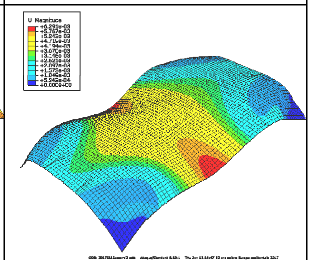
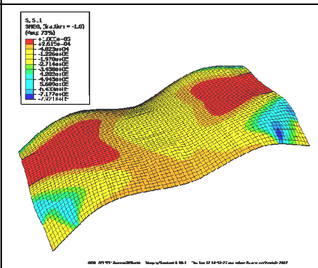
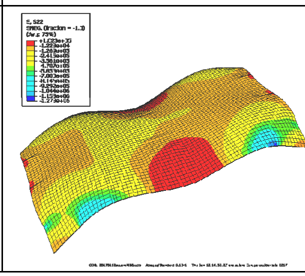
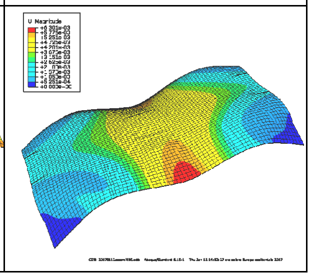
case	σ_{11} [N/m^2]	σ_{22} [N/m^2]	D[m]
1			
2			
3			
4			

Table 2. Results of the non linear analysis on the no-reinforced vault

2, due to the lower total load applied on the vault.

CASE 4, the analysis has not reached the convergence. It aborted for a load very little higher than case 3 and in detail for 51% of the live load applied. The stress state and the dis-

placements are almost equal to the previous case, with an increase of the tensile zone.
It can be concluded that the vault was able to support the self-weight, the weight of the infills and also a significant value of live loads, but it was not able to bear a no-symmetrical live load equal to 4 kN/m^2 , which was the value prescribed by the Italian rules for the new destination of the building. Therefore, different strengthening solution were applied to the described FE model, and they are illustrated in the following section.

4. STRENGTHENING OF THE VAULT

The consolidation proposals were aimed to stiffen the masonry vault and, at the same time, to comply the requests of the *Soprintendenza Archeologia, Belle Arti e Paesaggio per le province di Lodi, Cremona e Mantova,* who asked for a compatible and no-invasive strengthening. Four solutions were studied, all based on realizing *frenelli* in order to stiff the vault (Figure 5).
A non linear analysis was performed on the four models, considering only the load-case 4, which is the most heavy. The results, reported in Table 3, are given in terms of the two components of the stress state (σ_{11} and σ_{22}) and of the displacements.

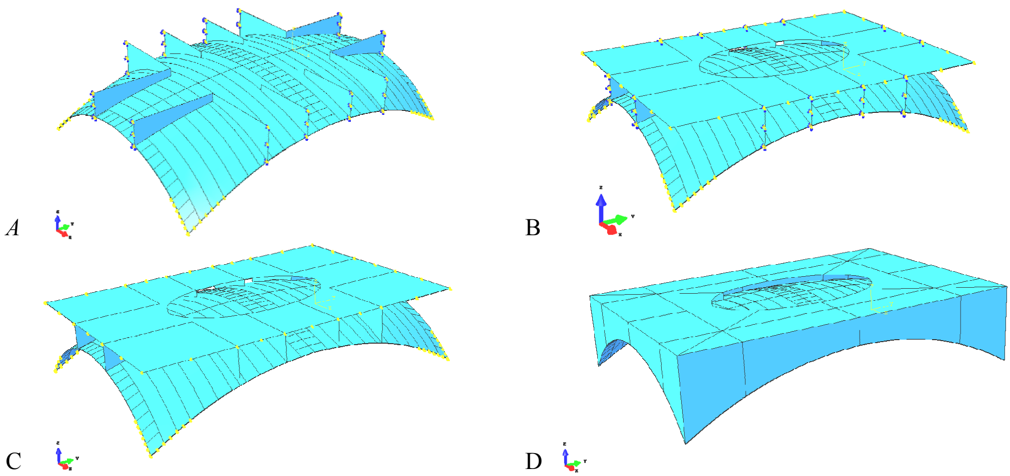


Figure 5. Strengthening solutions

4.1. Reinforced concrete *frenelli*

Solution A consists in realizing twelve reinforced concrete *frenelli,* 0.25 m thick*:* eight in transversal direction and four in longitudinal direction. Each *frenello* is connected to the vault. The connection could be realized by means of vertical metal connectors. The *frenelli* are also considered fixed to the perimeter walls, therefore fixed boundaries were assigned to the perimeter vertical edges. The constraints simulate the presence of horizontal connectors included in the concrete and inserted in the wall. It was found that the displacements are much lower than in the no reinforced vault: the maximum displacement is almost equal to 0.8 mm. However, the *frenelli* worked as a cantilever and the vault is hung up to them. Consequently, at the top of the *frenelli,* high concentrations of tensile stresses were found, which lead to a significant cracking of the concrete. With this solution, the resistance of the vault would be almost completely given to the metal horizontal connectors, but actually they cannot be perfectly fixed in the wall and, moreover, a stress concentration in this point can determine mechanism of local crisis.

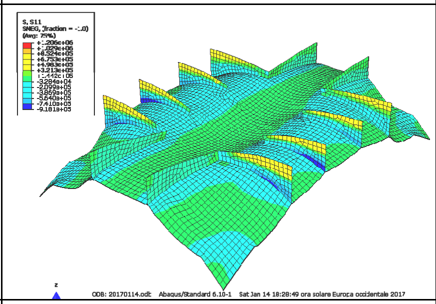
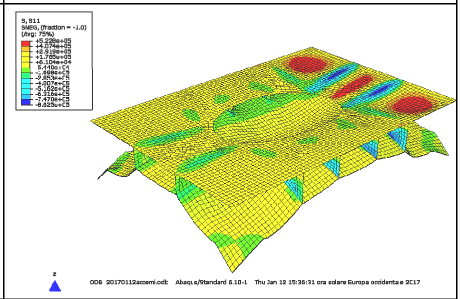
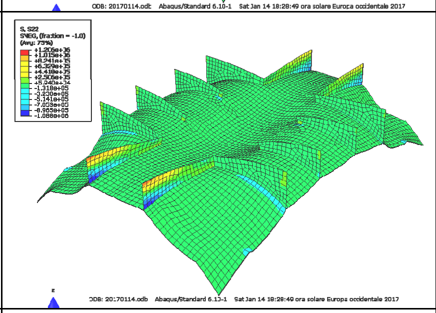
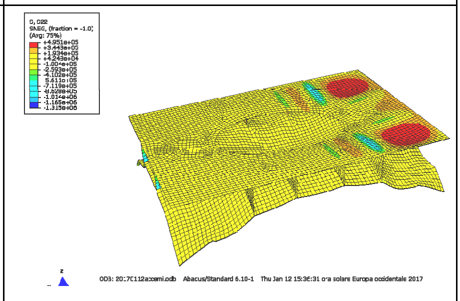
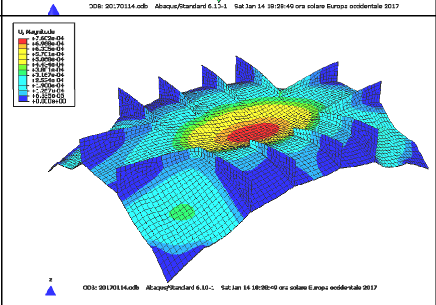
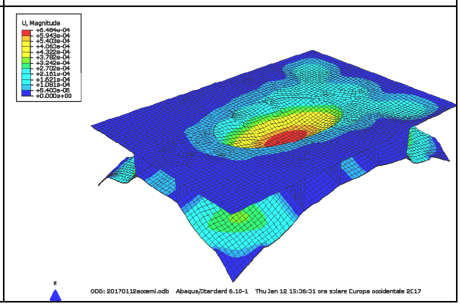
	A	B
σ_{11} $[N/m^2]$		
σ_{22} $[N/m^2]$		
D [m]		

Table 3. Results of the non linear analysis on the vault reinforced through concrete *frenelli* (solution A and B)

In order to decrease the tensile stresses at the top of the concrete *frenelli* a B solution was proposed, which includes a reinforced concrete slab above the *frenelli*. The slab was modeled connected to the *frenelli* and also fixed to the perimeter walls. Consequently, the slab works as a new connecting element, which leads the system to function as an integral whole, providing an advantageous contribution with respect to the earthquakes. Moreover, it is particularly useful to uniformly distribute the loads on the *frenelli*. The results of solution B shows that the vault was totally compressed and no tensile concentrations were found at the connection between *frenelli* and vault. Large tensile zones are visible on the slab, especially on the half loaded. However, the maximum value of the tensile stresses is still acceptable for reinforced concrete material. The B solution gives the further advantage of providing lower displacements: compared with the previous solution: the maximum displacement was found equal to 0.65 mm.

4.2. Masonry *frenelli*

The strengthening solution A and B described above improve the stress state of the vault and considerably increase its load-bearing capacity, until to support the loads prescribed by the Italian rules. However, the use of reinforced concrete elements for historical constructions is not always allowed, because it cannot be considered sufficiently respectful of the restoration principles such as the compatibility and the reversibility. Therefore, different solutions were studied, which adopt “masonry” instead of “concrete” for the *frenelli*.
Solution C changes solution B simply substituting the material of the *frenelli*. The constraints along the vertical perimeter edges were eliminated, taking into account that a fixed connection between perimeter walls and masonry *frenelli* is more difficult to realize than in the case of reinforced concrete. However, the reinforced concrete slab was still considered fixed to the walls. This solution did not lead to the convergence of the analysis, which aborted for a load almost equal to 75% of the live load applied.
By observing the stress state on the vault (Table 4, line 3), it can be seen that the masonry *frenelli* are mostly tense particularly at the connection with the slab. The maximum value of the tensile stresses in the masonry *frenelli* reach 0.16 MPa, which is much higher than the tensile strength attributed to the masonry. As a result, a significant damage state could occurr in the *frenelli*, causing the crisis of the consolidated system.
Then, a different position of the *frenelli* was evaluated (solution D). It was thought to better respond to the stress state and the deformed shape observed in the no-reinforced ribbed vault (Table 2 line 4). The stress state had shown a concentration of tensile stresses in the middle of the perimeter edges of the vaults. Then, four masonry *frenelli*, shaped as arches, were positioned along the four edges of the vault. Moreover, from the deformed shape, it was noted a significant raising at the four corners of the vault. Therefore, other four *frenelli* were added along the diagonals. They were interrupted at the center zone in order to avoid the increase of the total height of the consolidated vault. This new arrangement was completed with an upper reinforced concrete slab, which leads to the structural advantages described above. The analysis of this last solution provided very good results, both in terms of stress state and in terms of displacement (the maximum displacement is equal to 0.86mm). The vault and the masonry *frenelli*, are mostly compressed, except of small zone where the stresses are acceptable for masonry material. The significant tensile stresses are concentrated on the slab, but they are lower than the reinforced concrete strength, so the consequent damage does not lead to the crisis of the system.

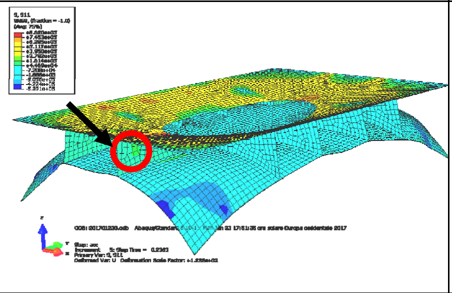
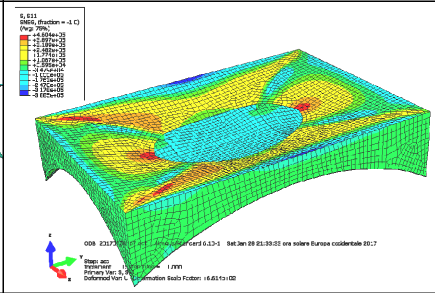
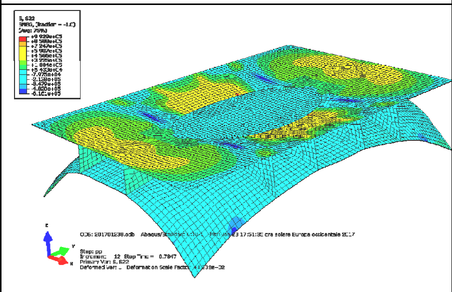
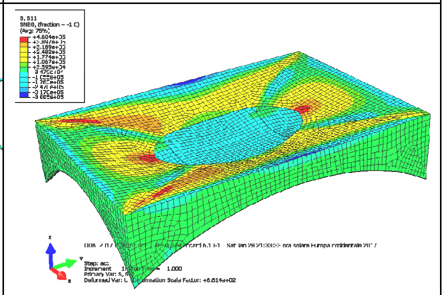
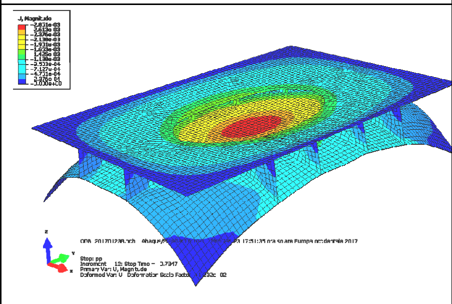
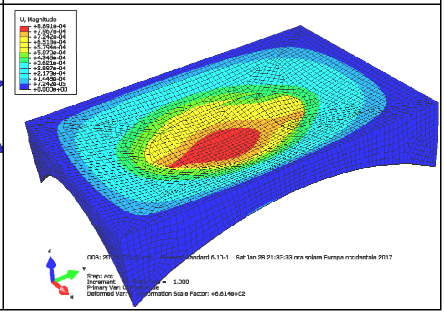
	A	B
σ_{11} [N/m^2]		
σ_{22} [N/m^2]		
D [m]		

Table 4. Results of the non linear analysis on the vault reinforced through masonry *frenelli* (Solution C and D)

5. CONCLUSIONS

The paper provides the structural analysis of a masonry ribbed vault. The need to adequate the vault to the loads prescribed by the Italian rules makes a strengthening intervention necessary. Different solutions were studied, based on a traditional system to consolidate vaults: the *frenelli*. It was found that this system is significantly improved by adding a reinforced concrete slab above, which makes the *frenelli* collaborative and, also, uniformly distributes the loads. Moreover, reinforced concrete *frenelli* and masonry *frenelli* were compared. It was found that both the solutions provide a safe structural behavior of the vault. However, the reinforced concrete *frenelli* , much stiffer than the vault, change its natural deformative state and, in addition, represent no removable and no compatible elements, according to the accepted restoration principles. By the opposite, it was found that masonry *frenelli*, correctly positioned to adjust the weaknesses of the no reinforced vault, and helped by the slab, can be an efficient and more respectful and compatible alternative for the consolidation of existing vaults.

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