NUMERICAL MICRO-MODELLING OF THE REINFORCED ARCH METHOD

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ABSTRACT

Brought to its maximum magnificence from the Roman genius, and then spread all over the World throughout History, the arch has become one of the most recognizable mark of our architectural legacy. The Reinforced Arch Method is a rather recent strengthening technique for masonry arches and vaults that entails the application of steel post-tensioned cables at their extrados or intrados, in order to enhance capacity and ductility. Many experimental tests have been performed on this technique, but numerical modelling has not been yet fully deepened. Indeed, by means of numerical finite element models, the present work mainly aims at confirming the effectiveness of this procedure. At the same time, it envisages calibrating and validating the numerical tools, by comparing their results with the ones derived from an experimental campaign on scale models, performed at the Polytechnic University of Milan, and the ones from limit analysis. Micro-models, made of linear-elastic blocks with hinges opening at their interfaces, have been produced and tested.

1. INTRODUCTION

1.1. The Reinforced Arch Method Strengthening Technique

The Reinforced Arch Method (RAM) is a rather recent strengthening technique, developed by Prof. Lorenzo Jurina at Polytechnic University of Milan (IT), based on the application of steel post-tensioned cables (Figure 1) at the extrados or the intrados of masonry arches and vaults (Jurina, 2012). This technique improves the response of the structure because of:

- Providing tensile resistance on one side of the structure, so as to contrast the opening of some hinges, preventing the full formation of the collapse mechanism.
- Applying an additional state of uniform compression that is able to re-center the thrust line and therefore increases the geometrical safety factor of the arch.
- Improving the resistance to shear failure between blocks (sliding), due to the increment of compression inside the structure.

All of these aspects have been proving, through both experimental tests and interventions on real structures, the great suitability of this procedure for the consolidation of historical masonry structures. In fact, tests have shown that both resisting capacity and ductility are significantly improved.

Figure 1. Examples of interventions through the RAM: a masonry arch at Villa Borromeo, Senago (MI), IT, (Jurina, 2003) and a ribbed vault in St. Caterina church in Lucca, IT, (Jurina, 2014).

The RAM can be interpreted as both a passive and an active strengthening technique. The first definition is due to the passive capability of providing extra strength, connected with the high tensile resistance of the steel cables. The second is due to the effect of post-tension, that induces a radial load confinement, proportional to the arch curvature (Figure 2). The active attitude of the RAM entails also the possibility to re-calibrate the post-tension in case of loss of tension, or when the conditions of the structure requires it.

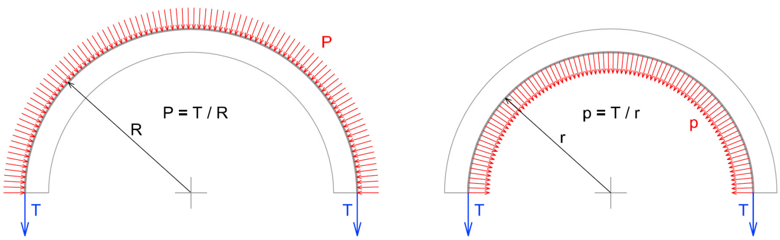

Figure 2. Confinement effect of the RAM.

This technique can be applied at the extrados or at the intrados of the arch. The first case is the most common one, since is the less aesthetically invasive when the visible part of the structure is the intrados, as in most of the cases (e.g. arch-bridges, church vaults, etc.). The second case, instead, is applied mostly when the extrados is not accessible (e.g. infilled vaults that need a deep dismantling to reach the extrados). When the cables are applied at the intrados, proper anchorage must be realized to transfer the radial load to the masonry structure, while with cables at the extrados the load is transferred by simple contact. The RAM, due to its low invasiveness and to its traditional materials usage, is a particularly compatible and reversible strengthening technique.

1.2. Experimental Campaign on Scale Models

The effectiveness of the RAM strengthening technique has been broadly proven by a great number of experimental tests. More than 500 tests on different scale models have been carried out by Prof. Jurina, Arch. Giglio and Arch. Bonfigliuoli, showing relevant improvements of the response of the arches.

The test campaign used in this paper for comparing numerical results concerns a wooden blocks, dry-joints roman arch scale model. The arch had a 1200 mm net span, 100 mm of depth and 100 mm of width. It was made of 77 beechwood elements, of density equal to 600 kg/m^3 (Figure 3). The vertical incremental load was applied by means of a wire connected to a steel device at the center of the voussoirs, at different fractions of the span.

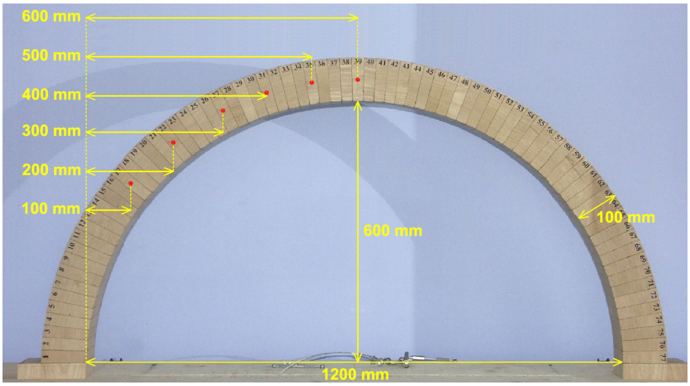

Figure 3. Semi-circular arch tested: points of load application and geometry. (Giglio, 2008).

The reinforcement was applied by means of two steel cables of 2 mm of diameter, positioned either at the extrados or at the intrados. Tests consisted on applying an external incremental load by means of controlled wire force. Two main types of application of the reinforcement were produced: one with sliding cables, therefore maintaining the tension constant during all the duration of the tests, and another with the cables anchored to the supporting table, after post-tensioning. Different levels of cable tension were tested (Table 1). Collapse always occurred due to the achievement of the maximum number of admissible hinges, with the consequent loss of equilibrium. In the case of sliding cables, results showed a relevant improvement of the load capacity of the arch, in both the reinforcement arrangements. In the anchored cables case, collapse was never reached, showing a dramatic increase of the capacity of the arch. In this second case, the loading phase was carried out by increasing of 392 N (40 kgf) stepwise the punctual load, until 2354 N (240 kgf), and then unloading with the same load step size. In every step, the amount of displacement in some significant points of the arch was measured, so as to enable the possibility to trace some load/displacements curves.

Cables force [N]	Sliding cables		Anchored cables
	Extrados	Intrados	Extrados
	Load positions (span fraction)	Load positions (span fraction)	Load positions (span fraction)
0	all	all	1/4, 1/2
20	all	-	-
39	all	-	-
59	all	-	-
79	all	-	-
98	all	all	1/4, 1/2
196	all	all	1/4, 1/2
294	all	all	1/4, 1/2

Table 1. Plan of the experimental tests on the semi-circular arch.

2. MODELS' FEATURES

To simulate the behavior of the experimental arch, the numerical tools used have been:
- The commercial program Ring (Limit State), for kinematic limit analysis.
- An Excel spreadsheet for graphic statics, including radial confinement load.
- Finite element micro-models, with the program Diana FEA 10.0.

Limit analysis has been first used to check the coherence of the experimental results with the theoretical ones. This preliminary study has permitted to discover that the effective depth of the tested arch was not the entire one. The different effective depths have been assessed to fit the experimental data, depending on the load position and the post-tension level applied, so as to make possible the comparison of results with the numerical models. Just a few representative cases have been numerically modeled, as reported in Table 2.

Case	Cable Position	Post-Tension [N]	Load Positions (span fraction)
Unreinforced	None	0	1/6, 1/2
Sliding Cables	Extrados	294	1/6, 1/2
	Intrados	294	1/6, 1/2
Anchored Cables	Extrados	0	1/2
	Intrados	0	1/2
	Extrados	294	1/2
	Intrados	294	1/2

Table 2. Cases chosen for numerical finite element analysis.

The micro-models produced, by means of Diana FEA, consisted on 2D plane stress elements forming the arch body. For the unreinforced models, 4-noded elements have been used, while for reinforced ones, 8-noded elements have been applied. Elements' side has been between 6 and 8 mm. In any case, each node presented 2 translational degrees of freedom. The cable reinforcement has been modeled using 2D cable-truss 3-noded elements, yet with 2 translational degrees of freedom for each node. The structure has been divided into 77 voussoirs, like the experimental arch. The contact interfaces between blocks, as well as the interface between the cable and the arch, have been modeled with quadratic interface elements (reinforced case) or linear ones (unreinforced case), to fit the plane elements of the arch. Translational supports have been applied at the imposts. Post-tension has been applied as pointed forces at the ends of the cable. External load has been applied as an incremental vertical pointed displacement. The loading stages have been:

1. Gravity load application (to unreinforced arch, 4 steps);
2. Addition of cables and post-tensioning (just in reinforced arch cases, 10 steps);
3. External loading (unreinforced and reinforced arch cases, until collapse).

Non-linear incremental analyses have been carried out, including in some cases non-linear geometry conditions, with 0.001 mm vertical displacement step. The iterative method that has been chosen was a full Newton-Raphson and convergence has been checked by means of energy, displacement and force, with 0.01 tolerance.

The arch body voussoirs have been modeled with linear elastic material, with parameters as faithful as possible to the ones of beechwood. The values chosen are 15000 MPa for the Young modulus and 0.3 for the Poisson's ratio. The density has been set as to maintain the overall weight of the original arch, which had 100 mm of depth and density of 600 kg/m^3.

The steel of the reinforcement cables has also been modeled with linear elastic material, with a Young modulus of 196000 MPa, Poisson's ratio of 0.3 and null density to neglect the cable weight.

Concerning interfaces, Coulomb friction behavior, with gap opening, has been chosen. The values of the parameters are reported in Table 3 and Table 4. For the interfaces between blocks a high value of the friction angle has been fixed, to avoid any sliding. The elastic parameters have been estimated reasonably comparing the ones concerning masonry (Lourenço, 2002), in absence of more detailed data. Concerning cable/arch interface, the value of the friction angle is not fixed, since a sensitivity analysis, presented in the next chapter, has been performed to assess the most suitable value to apply. All the other parameters have been reasonably estimated.

Blocks Interface - Parameters				
	Parameter	Notation	Value	Unit
Linear	Normal stiffness	K_n	10000	N/mm^3
	Shear stiffness	K_s	2000	N/mm^3
Friction	Cohesion	c	0	MPa
	Friction angle	φ	89°	Deg
	Dilatancy angle	ψ	0	rad
Gap opening	Tensile strength	f_t	0	MPa

Table 3. Coulomb friction behavior parameters of blocks' interfaces.

Steel Cable - Geometric Parameters				
Parameter		Notation	Value	Unit
Cable Diameter		d	2	mm
Cables Section Area		A	6.18	mm^2
Friction Surface Width		t	0.1	mm
Cable/Arch Interface - Parameters				
	Parameter	Notation	Value	Unit
Linear	Normal stiffness	K_n	10000	N/mm^3
	Shear stiffness	K_s	100	N/mm^3
Friction	Cohesion	c	0	MPa
	Friction angle	φ	0.6° - 30°	Deg
	Dilatancy angle	ψ	0°	Deg
Gap Opening*	Tensile strength	f_t	0	MPa

Table 4. Geometric and friction parameters of cable/arch interface. *In the case of reinforcement at the intrados gap opening has not been enabled.

3. UNREINFORCED ARCH - RESULTS

Results concerning the unreinforced arch loaded at 1/6 and1/2 of the span are presented. Both linear and non-linear geometry analysis have been performed. As it can be noticed, the capacity curves obtained (Figures 4 and 6), the final value of capacity and the collapse hinged configurations (Figures 5 and 7) fit the expected values computed with limit analysis, and even the experimental ones (Table 5). The fact that limit analysis capacity is not equal to the experimental one, despite the back-analysis previously performed on the arch depth to fit its value, is only due to approximation on the calculation of the equivalent depth itself. Notice that in the case of non-linear geometry softening occurs, while for linear geometry the final trend of capacity curve is flat.

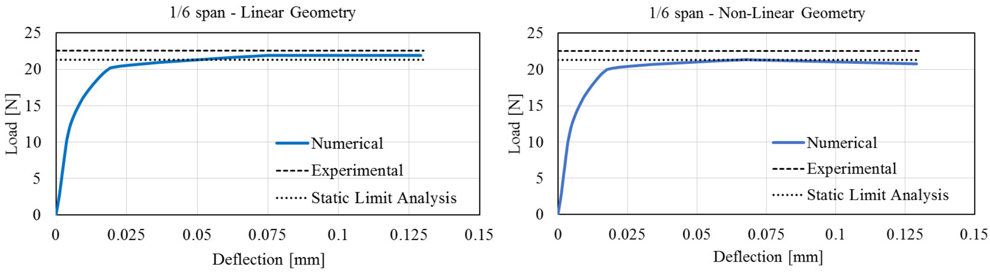

Figure 4. Micro-model capacity curves of the unreinforced arch, loaded at 1/6 of the span.

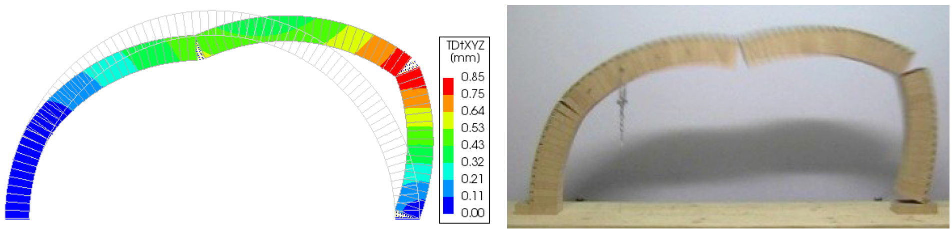

Figure 5. Micro-model unreinforced arch, loaded at 1/6 of the span. Comparison of deformation (x200 amplification) with the experimental one at collapse.

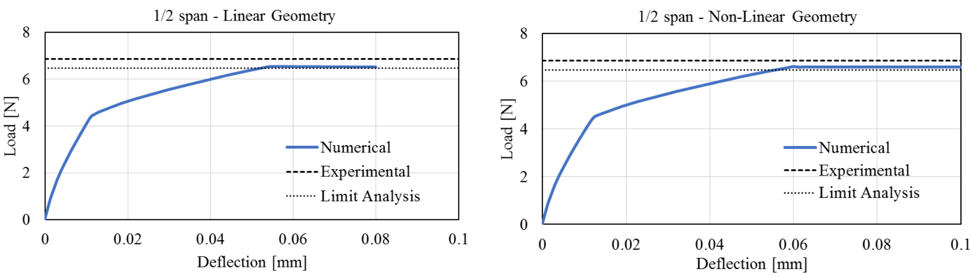

Figure 6. Micro-model capacity curves of the unreinforced arch, loaded at 1/2 of the span.

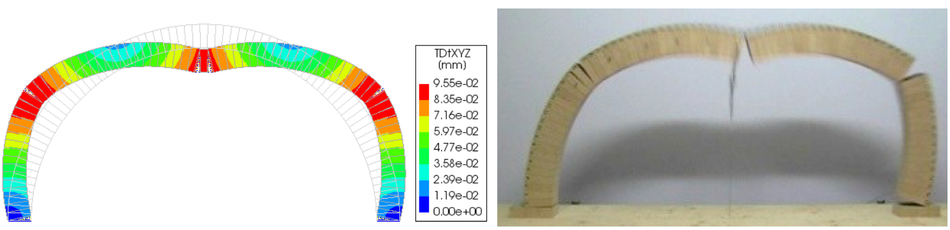

Figure 7. Micro-model unreinforced arch, loaded at 1/2 of the span. Comparison of deformation (x1000 amplification) with the experimental one at collapse.

Load position (span fraction)	Non-Linear Geometry	Collapse load [N]			Error	
		Numerical	Limit Analysis	Experimental	To Limit Analysis	To Experimental
1/6	No	21.9	21.3	22.6	3%	-3%
1/6	Yes	21.3	21.3	22.6	0%	-5%
1/2	No	6.5	6.5	6.9	1%	-5%
1/2	Yes	6.6	6.5	6.9	2%	-4%

Table 5. Unreinforced arch results, compared to limit analysis and experimental ones.

4. REINFORCED ARCH - RESULTS

4.1. Friction Angle Sensitivity Analysis

Since friction angle between cable and arch was unknown, its effects on the sliding behavior and on the capacity have been investigated. 4 values have been considered: 0.6°, 5°, 20° and 30°. For the sake of brevity, only the case loaded at half span with cables at the extrados has been studied.

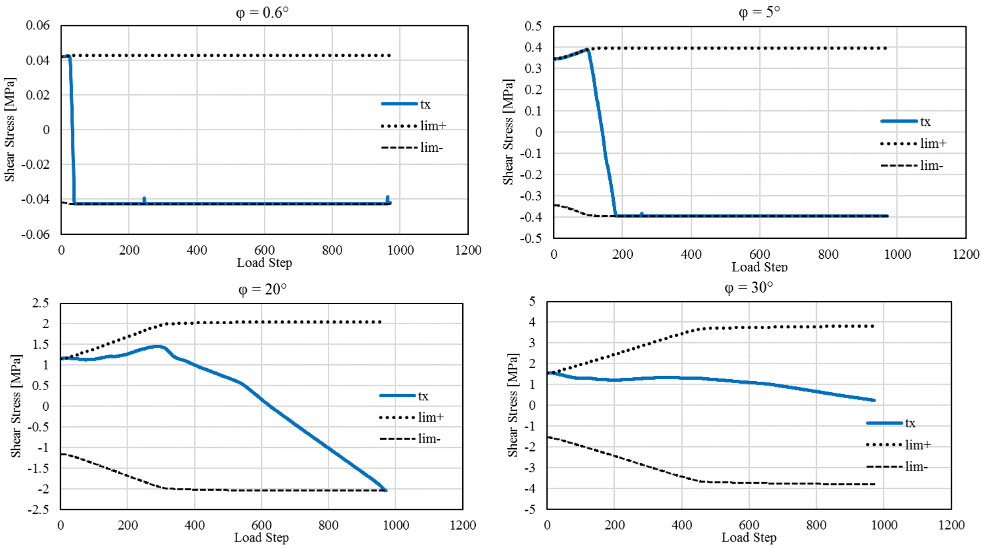

Figure 8. Shear stress at cable/structure interface in a 45° point of the arch during external loading until collapse.

In Figure 8 the trends of shear stress at interface (in a representative point at 45°) are presented. It can be noticed that for low values of φ adherence is almost always present, except for a short period of inversion of the sign, while for higher values adherence is achieved. In

Figure 9 a comparison between capacity obtained with the different values of friction angle is proposed. The curve that best fits the experimental value is the one with the lowest φ. Therefore a value of 0.6° has been kept for further analysis, implying sliding conditions between cable and arch.

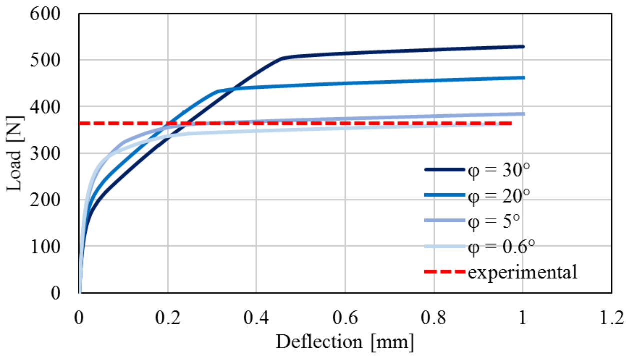

Figure 9. Capacity curves obtained with different values of the friction angle between cable and arch.

4.2. Free Sliding Cables – Results

The cases concerning reinforced arch reported in Table 2 have been modeled. Results are shown in Figures 10 and 12, and Table 6. Linear geometry has been used. Also in this case, capacity curves fit the expected values, especially for the case loaded at 1/2 of the span, whose geometry has been better calibrated to fit the experimental results. The value of cable tension at collapse shows a rather constant value, equal to the initial one of 294 N (Figures 11, 13).

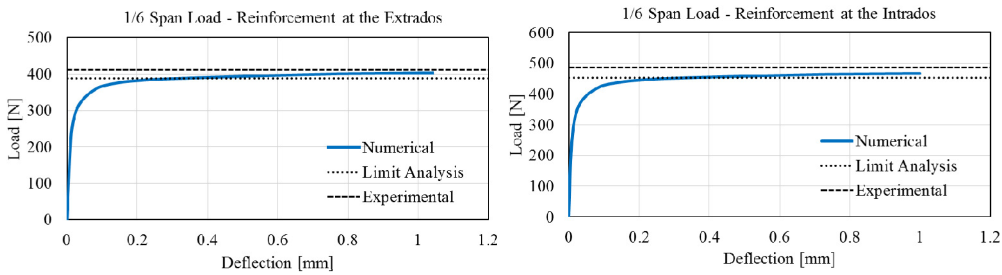

Figure 10. Micro-model capacity curves of the reinforced arch with sliding cables, loaded at 1/6 of the span.

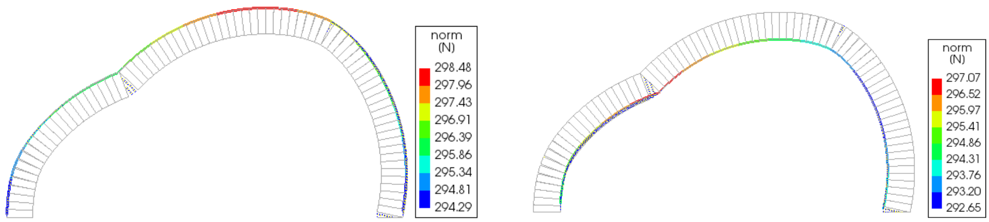

Figure 11. Micro-model reinforced arch with sliding cables, loaded at 1/6 of the span. Cable tension at collapse deformed configuration (x50 amplification).

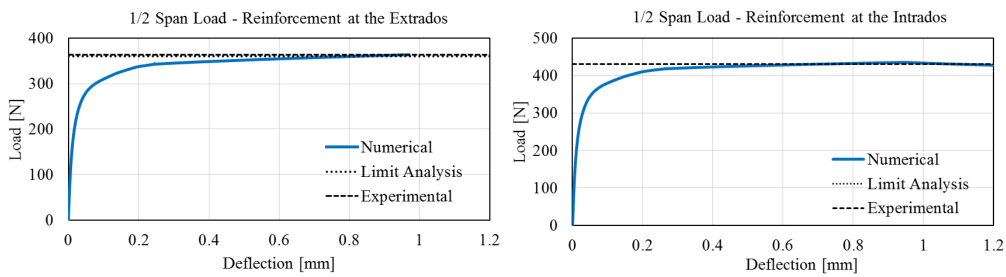

Figure 12. Micro-model capacity curves of the reinforced arch with sliding cables, loaded at 1/2 of the span.

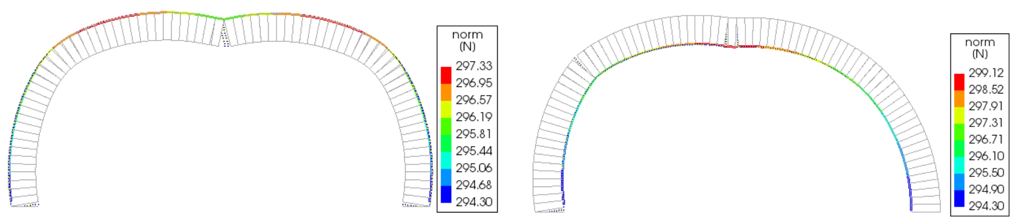

Figure 13. Micro-model reinforced arch with sliding cables, loaded at 1/2 of the span. Cable tension at collapse deformed configuration (x70 amplification).

Load position (span fraction)	Collapse load [N]			Error to Numerical	
	Numerical	Limit Analysis	Experimental	Limit Analysis	Experimental
1/6	468	452	486	3.5%	-3.7%
1/6	403	389	411	-1.9%	3.8%
1/2	436	431	431	1.2%	1.2%
1/2	363	360	364	-0.2%	0.9%

Table 6. Reinforced arch results, compared to limit analysis and experimental ones.

4.3. Anchored Cables – Results

The cases reported in Table 2, concerning anchored cable models, have been modeled. Only capacity has been analyzed. Both linear and non-linear geometry have been considered. In Figure 14 it can be noticed that the experimental dramatic increase of the capacity is confirmed. In the linear geometry case, no collapse is reached, and the curve follows a linear trend. In the non-linear geometry case, instead, the curve is quasi-linear, showing collapse due to buckling.

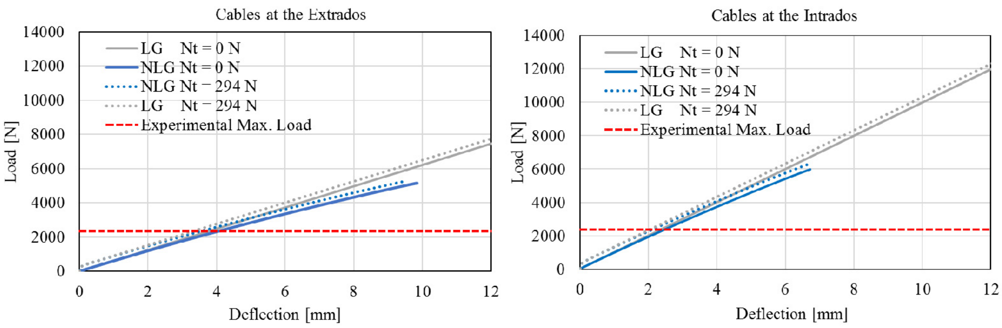

Figure 14. Micro-model capacity curves of the reinforced arch with anchored cables, loaded at 1/2 of the span.

The difference in the behavior induced by post-tension in this case is minimum, since the cable traction is so relevantly increasing during external loading that the post-tension initial force becomes negligible (Figure 15).

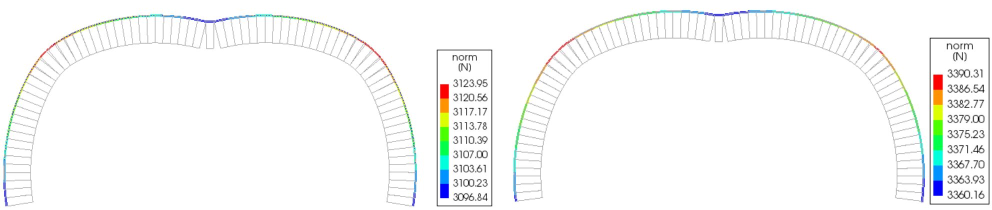

Figure 15. Micro-model of the reinforced arch with anchored cables, loaded at 1/2 of the span. Cable tension at 6 mm of deflection of the central block: no post-tension case (left), post-tensioned cable case (right). Deformed configuration (x10 amplification).

5. CONCLUSIONS

The present work aimed at confirming the effectiveness of the RAM strengthening technique, using finite element numerical models. Experiments on collapse of wooden blocks scaled roman arches have been numerically reproduced and simulated, through micro-models. The validity of the results obtained have been confirmed by comparison both with

the experimental and the limit analysis results. After a calibration phase, by means of limit analysis, to compensate geometrical problems connected with the reduction of the effective cross-section of the arch, numerical analyses have been performed. Results showed good accordance with experimental and limit analysis ones. Moreover, a more detailed analysis has been performed to investigate the influence of friction angle between cables and arch on the global behavior and the capacity of the structure. This analysis has underlined that the transition from sliding to adherence of cables was governed by the value of the friction angle. Greater value of this parameter has shown greater overall capacity. Also, the cases of anchored cables have been studied, confirming the dramatic increase of capacity highlighted in the experimental campaign, if compared with the cases of not-anchored ones.
These numerical tools can be further enhanced in terms of material definition of both the arch body and the reinforcement, especially for simulating the behavior of real masonry arches. Possible future improvements can be carried out by assigning a non-linear material at the arch blocks, to capture damage in compression. Also, non-zero tensile strength at the interfaces (mortar joints) can be added. Concerning the modelling of the reinforcement cables, a yield criterion of steel can be added, to capture possible post-elastic effects, in particular for obtaining a reliable capacity curve in the cases of anchored cables. Finally, non-fixed anchors can be introduced, to simulate potential displacements during the arch loading. All this treatise could be extended to barrel vaults, by means of 3D models, to investigate the effect of diffusion of stresses in the third dimension, and perform a parametrical analysis for the optimization of the strengthening technique.

REFERENCES

Giglio, S. M.: Consolidamento di archi e volte in muratura mediante la tecnica dell’arco armato. Approccio Sperimentale. Diss. Tesi di Laurea, rel. Lorenzo Jurina, Politecnico di Milano A.A. 2007-2008, 2008.

Bonfigliuoli, S.: Consolidamento strutturale e antisismico di archi e volte in muratura : una sperimentazione sulla tecnica dell'arco armato. Diss. Tesi di Laurea, rel. Lorenzo Jurina, Politecnico di Milano A.A. 2010-2011, 2011.

Jurina, L.: Il consolidamento strutturale della chiesa di Santa Caterina in Lucca. 2014.

Jurina, L.: La possibilità dell'approccio reversibile negli interventi di consolidamento strutturale. *Atti del XIX Convegno Scienza e Beni Culturali La reversibilità nel restauro. Riflessioni, Esperienze, precursor di ricerche*, Bressanone, 2003, pp. 1-4.

Jurina, L.: Strengthening of Masonry Arch Bridges with “RAM”–Reinforced Arch Method. *Seminario IABMAS,* Stresa (VB), 2012.

Lourenço, P. B.: Computations on Historic Masonry Structures. *Progress in Structural Engineering and Materials*, 4.3, 2002, pp. 301-319.